3. JOSEPH D., Stability of Fluid Motions, Mir, Moscow, 1981.
4. YIH CH.-SH., Note on eigenvalue bounds for the Orr-Sommerfeld equation, J. Fluid Mech., 38, 2, 1969.
5. KOCHAR J.T. and JAIN R.K., Note on Howard's semicircle theorem, J. Fluid Mech., 91, 3, 1979.
6. MAKOV YU.N. and STEPANYANTS YU.A., On growing wave parameters in shear flows. Okeanologiya 23, 3, 1983.
7. MAKOV YU.N. and STEPANYANTS YU.A., Note on the paper of Kochar and Jain on Howard's semicircle theorem. J. Fluid Mech., 140, 1984.
8. MAKOV YU.N. and STEPANYANTS YU.A., On the influence of profile curvature on growing wave parameters in shear flows, Okeanologiya, 24, 4, 1984.

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THE PASSAGE OF A NON-STATIONARY PULSE THROUGH A LAYER WITH DAMPING*

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The one-dimensional problem of the passage of a non-stationary stress pulse through an acoustic layer possessing internal friction is examined. The damping in the layer is described by the model of a Voigt medium /1/. The use of a Laplace transformation in time reduces the problem to the evaluation of a certain contour integral. The integrand has a denumerable number of poles and one essential singular point in the complex plane. It is proved that the integral under consideration can be evaluated in the form of a series of residues of the integrand.

1. Let a stress pulse $\sigma_z = p_0(t),\ z = 0$ be incident on a layer $0 \leqslant z \leqslant h$. We consider the layer to be a solid body possessing internal friction. The Voigt model /2/

$$\sigma_z = \lambda u' + \eta u'^{\cdot} \tag{1.1}$$

is the standard model for internal friction for acoustic wave propagation, where $u = u_z(z, t)$ is the displacement λ is the elastic modulus η is the viscosity, and differentiation with respect to time is denoted by a dot and with respect to the coordinate z by a prime. Adding the equation of motion $\rho u^{\cdot\cdot} = \sigma_z'$, to (1.1) we arrive at an equation in the function u

$$\rho u^{\cdot\cdot} = \lambda u'' + \eta u''^{\cdot} \tag{1.2}$$

To fix our ideas, we consider the opposite face of the layer stress-free. Then the boundary conditions have the form

$$\lambda u' + \eta u'^{\cdot} = \begin{cases} p_0(t), & z = 0 \\ 0, & z = h \end{cases} \tag{1.3}$$

For simplicity we consider the initial conditions to be zero $u = u^{\cdot} = 0,\ t = 0$.

Applying a Laplace transformation in time to the relationships (1.2) and (1.3), we obtain for the most interesting characteristic, namely, the rate of displacement of the face $z = h$

$$v(t, h) = \frac{1}{2\pi i\rho} \int\limits_{\delta - i\infty}^{\delta + i\infty} \frac{P_0(s)\, e^{st} ds}{\operatorname{sh}(hs/\gamma)},\quad \gamma = \gamma(s) = \sqrt{c^2 + \varepsilon s},\quad \delta > 0 \tag{1.4}$$

$$c = \sqrt{\lambda/\rho},\quad \varepsilon = \eta/\rho$$

Here c is the speed of sound, and $P_0(s)$ is the Laplace transform of the function $p_0(t)$.

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2. The integrand in (1.4) has the essential singular point $s = \alpha = -c^2/\varepsilon$. This distinguishes the mathematical properties of the problem under consideration from the properties of problems based on other damping models /1/. The essential feature of the distinction is the following. A broad class of damping models that are classical /1/ is described by Volterra integral operators of the second kind with exponential kernels. The Laplace transforms of such kernels are single-valued analytic functions in the complex plane. In contrast, the Voigt model that is standard in acoustics is described by a Volterra integral operator of the first kind /1/ that results in the presence of an essential singular point for the integrand in (1.4) in the problem under consideration. We will show that this singularity does not hinder the evaluation of the integral (1.4) in the form of a series in residues.

For simplicity, we will consider $p_0(t)$ to act only in a finite time interval $0 \leqslant t \leqslant T$. Then $P_0(s)$ is an entire function of exponential type, where the estimate

$$P_0(s) \leqslant Ae^{|s|T} \quad (A = \mathrm{const}) \tag{2.1}$$

holds, permitting, say, deformation of the original contour L into the contour L_1 (figure) enclosing the essential singular point α and the negative poles s_k for $t > T$. The residues at the poles in the domain between the old and new contours are certainly taken into account here.

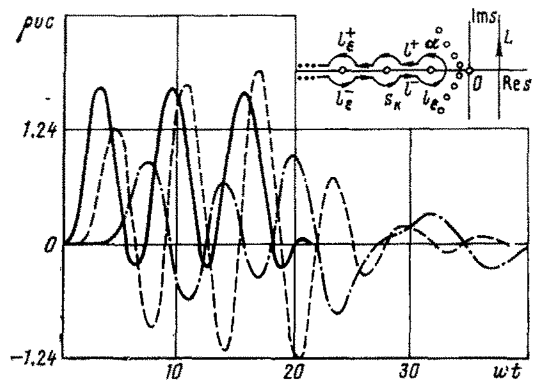
Let us separate the contour L_1 into a contour l_ε (an arc of radius ε) enclosing the point α and a set of contours of the type l^+, l^- and $l_\varepsilon^+, l_\varepsilon^-$. The arcs of the semicircles l_ε^+ and l_ε^- of radius ε enclose the negative poles s_k while the contours l^+ and l^- proceed along the edges of the real axis. The validity of the relationships

$$\left(\int\limits_{l^+} + \int\limits_{l^-}\right) ds = 0, \quad \lim \int\limits_{l_\varepsilon \pm} ds = \frac{1}{2}\,\mathrm{Res}\,(s_k), \quad \lim \int\limits_{l_\varepsilon} ds = 0 \quad (\varepsilon \to 0) \tag{2.2}$$

is established by direct substitution.

It follows from (2.2) that the original integral in (1.4) can be evaluated in the form of a series in residues of the integrand.

For $t < T$ analogous manipulations are performed by using a specific form of the function $p_0(t)$.



A numerical realization shows that the method proposed shortens the calculation time by a factor of approximately 10^2 as compared with direct evaluation of the integral (1.4). The passage of a sinusoidal pulse in three wave-lengths through a layer of different thickness is displayed in the figure. The solid lines correspond to a $1/4$ wavelength thickness, the dashes to a $1/2$ wavelength thickness, and the dash-dot line to unit wavelength. Damping in the layer is $\varepsilon\omega/c^2 = 0.39$, which corresponds to certain polymer composites. There is a tendency for a decrease in the amplitude of the fundamental osillations (the first three crests of the wave) as the layer thickness grows. This is natural and is explained by the influence of the damping. The process is further characterized by a gradual approach of the velocity $v(t, h)$ to zero, where the decrease of the function $v(t, h)$ occurs more rapidly for a thin layer. At first glance, this paradoxical fact is explained by the following. The pulse transmitted into the medium generates natural modes of layer oscillation. The fundamental natural frequency decreases as the layer thickness grows, and lower frequencies damp out more weakly than higher ones.

REFERENCES

1. RABOTNOV YU.N., Elements of the Hereditary Mechanics of Solids. Nauka, Moscow, 1977.
2. LANDAU L.D. and LIFSHITS E.M., Theory of Elasticity, Nauka, Moscow, 1987.

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